Regularity study of function in inhomogeneous Besov Spaces and the Riemann's series (Poster)

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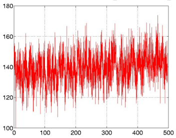
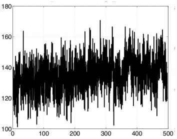
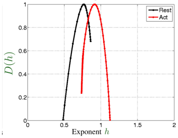
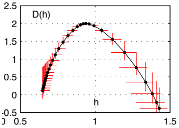
We have, from real data analysis, the following multifractal spectrum estimate









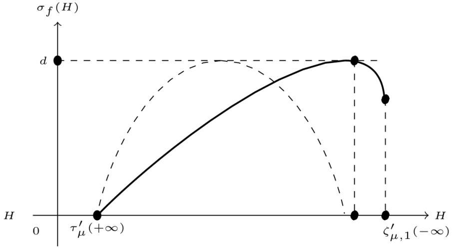
The idea is then to change the framework and space used from

$$f \in W^{s,p}(\mathbb{R}^D) \quad \text{with } |f|_{W^{s,p}(\mathbb{R}^D)} := \left(\iint_{\mathbb{R}^{2D}} \frac{|\Delta_h^n f(x)|^p}{|h|^{sp+D}} dxdh \right)^{1/p} < +\infty.$$

with affine multifractal spectrum to

$$f \in W^{\mu,p}(\mathbb{R}^D) \quad \text{with } |f|_{W^{\mu,p}(\mathbb{R}^D)} := \left(\iint_{\mathbb{R}^{2D}} \frac{|\Delta_h^n f(x)|^p}{\mu(B[x, x+nh])^p |h|^{2D}} dxdh \right)^{1/p} < +\infty.$$

with $\mu = \mu_t \otimes \mu_a$ (μ_t and μ_a will be supposed equal here) where generic function have concave multifractal spectrum.



For $0 < d < D$ and $d' := D - d$, my goal is to look at

$$f_a := f|_{\mathcal{H}_a} : \begin{array}{ccc} \mathbb{R}^d & \longrightarrow & \mathbb{R} \\ t & \longmapsto & f(t,a). \end{array}$$

with $\mathcal{H}_a := \{(t,a) \mid t \in \mathbb{R}^d\}$ the d-dimensional affine subspace of $\mathbb{R}^D$ for $a \in \mathbb{R}^{d'}$ (here $d = d'$).

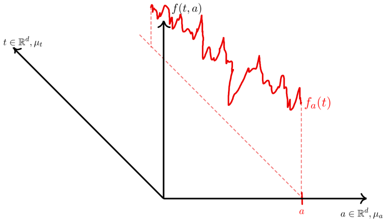


Figure: Representation of f_a for $a \in \mathbb{R}^d$ in $\mathbb{R}^{2d}$ with measure $\mu = \mu_t \otimes \mu_a$

The multifractal spectrum $\mathcal{D}_{f_a}(h) = \dim_H(E_{f_a}(h))$ highly depend on the trace in $a \in \mathbb{R}^d$, contrarily to what happens for standard Soboloev and Besov space with

$$E_{f_a}(h) = \{x \in \mathbb{R}^d \ : \ h_{f_a}(x) = h\}.$$

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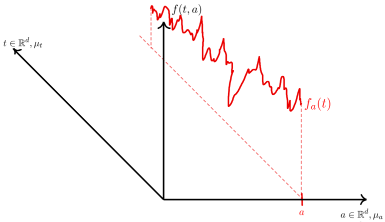

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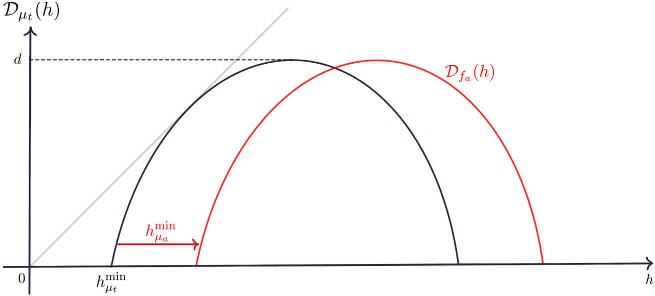

Figure: Multifractal spectrum $\mathcal{D}_{f_a}$ deduced from $\mathcal{D}_\mu$

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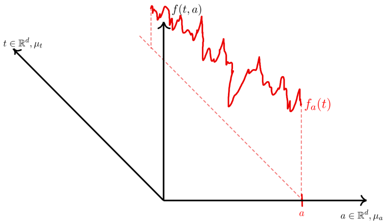

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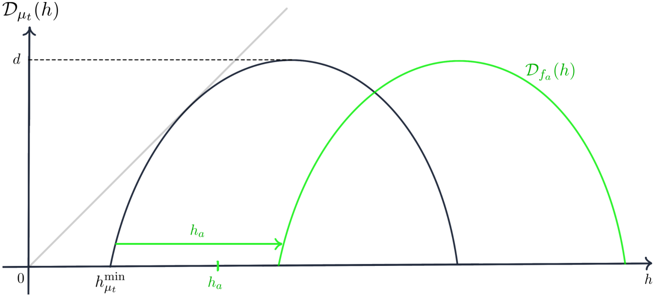

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